Aircraft Dynamics Identification

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Introduction

Aircraft dynamics identification has been a long-standing problem in aircraft engineering and is essential today for the optimization of flight trajectories in aircraft operations. This motivates the search for accurate dynamical systems identification techniques, the main topic of this study. The application we are most interested in here is aircraft fuel consumption reduction. This is a major goal for airlines nowadays, mainly for economic reasons, but also because it implies less CO_2 emissions. We limit our study to civil flights, and more specifically to the climb phase, where we expect to have more room for improvement. The techniques presented hereafter are suited for data extracted from the Quick Access Recorder (QAR). They contain multiple variables such as the pressure altitude and the true airspeed, with a sample rate of one second.

Aircraft dynamics

The main flight mechanics model used in this study is the following:

$$\begin{cases} \dot{h} = & V \sin\gamma, & (1) \\ \dot{V} = & \dfrac{T\cos\alpha - D - mg\sin\gamma}{m}, & (2) \\ \dot{\gamma} = & \dfrac{T\sin\alpha + L - mg\cos\gamma}{mV}, & (3) \\ \dot{m} = & -C_{sp}T, & (4) \end{cases}$$

In system (1)-(4), the elements T, D, L and C_{sp} are unknown and assumed to be functions of the state variables $\boldsymbol{x} = (h, V, \gamma, m)^\top$ and control variables $\boldsymbol{u} = (\alpha, N_1)^\top$.

Table 1: Flight mechanics nomenclature

h Altitude	V Speed	α A. of attack
γ Path angle	m Mass	N_1 Fan speed
T Thrust	C_{sp} Sp. cons.	D, L Drag-Lift

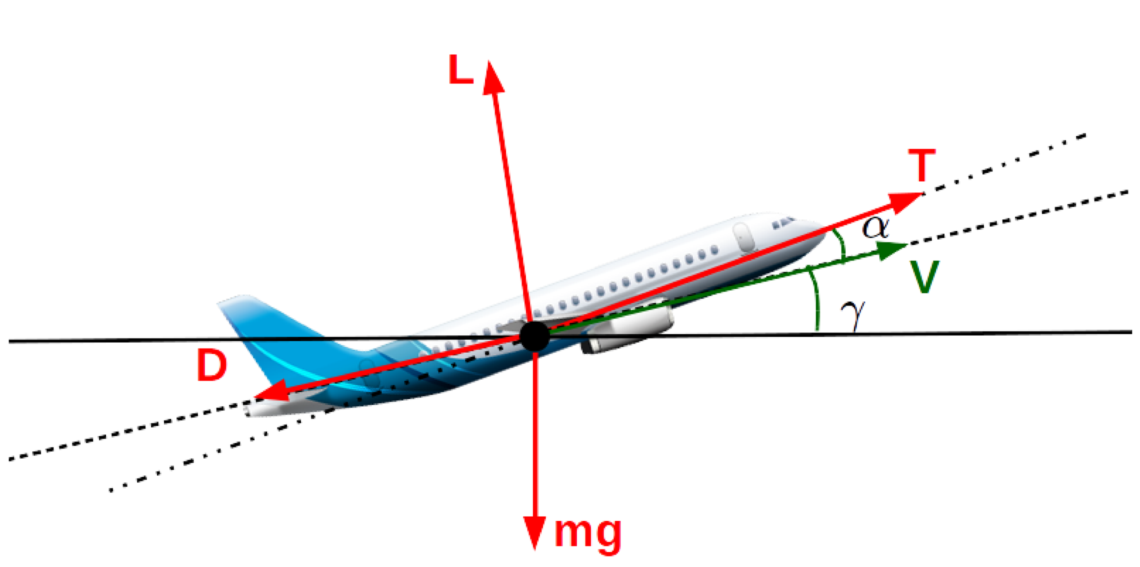

Figure 1: Flight mechanics standards

Objectives and constraints

- Estimate custom dynamics $\dot{\boldsymbol{x}} = g(\boldsymbol{x}, \boldsymbol{u})$ for individual A/C with good accuracy;
- Use QAR data;
- Leverage flight mechanics knowledge;
- Use parametric methods only, for easier model interpretation.

Feature engineering

Based on flight mechanics knowledge, new atmospheric variables can be derived from the state $\boldsymbol{x}$ and control $\boldsymbol{u}$ variables: SAT, ρ, M. Possible dependencies on all available variables were determined for each unknown function:

$$\begin{cases} T & \text{function of } M, \rho, N_1, \\ D & \text{function of } M, \rho, \alpha, V, \\ L & \text{function of } M, \rho, \alpha, V, \\ C_{sp} & \text{function of } M, h, SAT, \end{cases} \quad (5)$$

and linear models are assumed for all of them:

$$\begin{aligned} T &= X_T \cdot \boldsymbol{\theta}_T, & C_{sp} &= X_{csp} \cdot \boldsymbol{\theta}_{csp}, \\ D &= X_D \cdot \boldsymbol{\theta}_D, & L &= X_L \cdot \boldsymbol{\theta}_L. \end{aligned} \quad (6)$$

For increased flexibility, monomial expansions of the variables from (5) were generated and feature selection using the *Bolasso algorithm* [1] was performed to build X_T, X_{csp}, X_D and X_L.

Multi-task regression problem

Dropping equation (1) (which does not contain any unknown element) and leaving only the functions to be estimated in the r.h.s of the remaining equations leads to the following set of regression models

$$\begin{cases} Y_1 = X_1 \cdot \boldsymbol{\theta}_1 + \varepsilon_1, & (7) \\ Y_2 = X_2 \cdot \boldsymbol{\theta}_2 + \varepsilon_2, & (8) \\ Y_3 = (X_T \cdot \boldsymbol{\theta}_T)(X_{csp} \cdot \boldsymbol{\theta}_{csp}) + \varepsilon_3, & (9) \end{cases}$$

where

$$Y_1 = m\dot{V} + mg\sin\gamma, \qquad Y_2 = mV\dot{\gamma} + mg\cos\gamma,$$
$$Y_3 = C, \qquad \boldsymbol{\theta}_1 = \begin{bmatrix} \boldsymbol{\theta}_T \\ \boldsymbol{\theta}_D \end{bmatrix}, \quad \boldsymbol{\theta}_2 = \begin{bmatrix} \boldsymbol{\theta}_T \\ \boldsymbol{\theta}_L \end{bmatrix},$$
$$X_1 = \begin{bmatrix} X_T\cos\alpha \\ -X_D \end{bmatrix}, \quad X_2 = \begin{bmatrix} X_T\sin\alpha \\ X_L \end{bmatrix},$$

and the random variables $\varepsilon_1, \varepsilon_2, \varepsilon_3$ account for the model and data noise.

While T appears in (7)-(9), note that D, L and C_{sp} take part in a single equation each. Equation (9) is clearly the problematic one, because of the presence of a product between the unknowns C_{sp} and T. This means that we do not have linearity on the parameters, but also that this regression cannot be solved to determine both elements separately: only the product of them is *identifiable* here. These obstacles led us to tackle the regression problems together, in a *multi-task* framework [2]:

$$Y = f(X, \boldsymbol{\theta}) + \varepsilon, \quad (10)$$

where

$$\begin{aligned} Y = [Y_1, Y_2, Y_3]^\top, \quad \varepsilon = [\varepsilon_1, \varepsilon_2, \varepsilon_3]^\top \in \mathbb{R}^3, \\ X = [X_T, X_{csp}, X_L, X_D]^\top, \\ \boldsymbol{\theta} = [\boldsymbol{\theta}_T, \boldsymbol{\theta}_{csp}, \boldsymbol{\theta}_L, \boldsymbol{\theta}_D]^\top. \end{aligned} \quad (11)$$

The main idea here is that multi-task learning allows us to share the same thrust function T between all tasks. By doing so, we expect that some information gathered while learning tasks (7) and (8) will be transferred to task (9) during the process. This should help to reduce the non-identifiability issue, while improving prediction accuracy as already seen in other *transfer learning* applications [3, 2].

Results

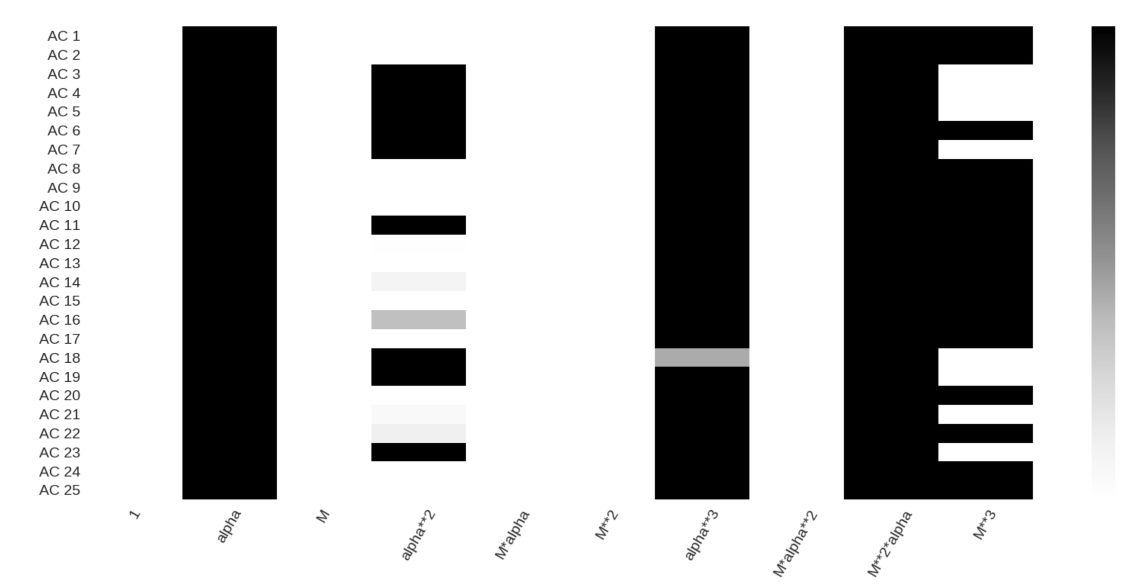

Figure 2: Bolasso selection for L features of 25 similar A/C. Each row corresponds to a different A/C and each column to a different possible feature. The cells color indicates the selection frequency over 128 *Lassos* solved. Training was performed using 10 471 302 observations in total.

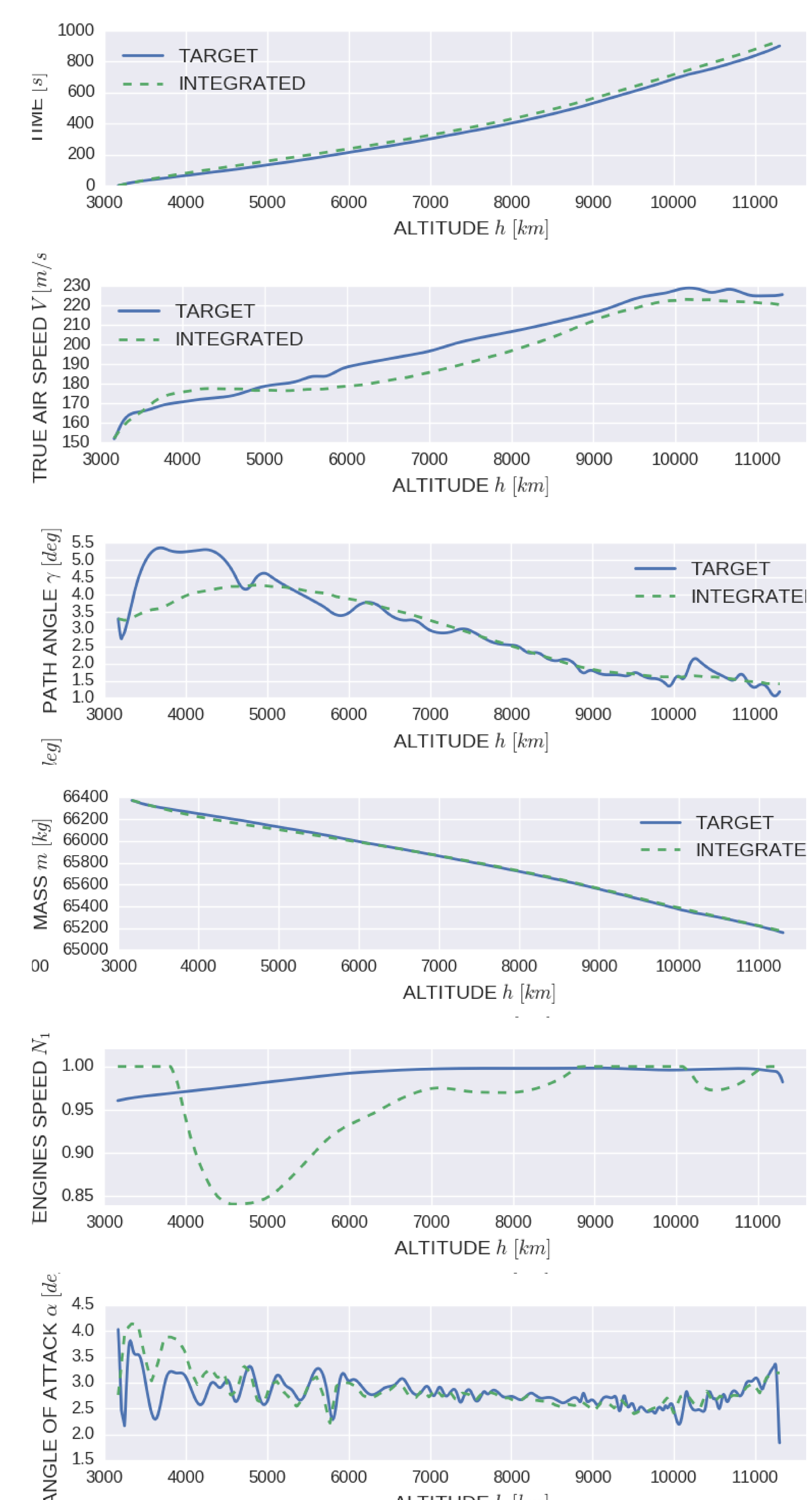

Figure 3: Flight resimulation of identified dynamics using BOCOP [4]. Training was performed using Nonlinear Least-Squares to solve (10) with real data from other flights performed by the same A/C.

References

[1] F. Bach.
Bolasso: model consistent Lasso estimation through the bootstrap.
ICML, pages 33–40, 2008.

[2] R. Caruana.
Multitask learning.
Machine Learning, 28(1):41–75, 1997.

[3] T. Evgeniou, C. A. Micchelli, and M. Pontil.
Learning multiple tasks with kernel methods.
J. Mach. Learn. Res., 6:615–637, 2005.

[4] Inria Saclay Team Commands.
Bocop: an open source toolbox for optimal control, 2017.