Motivation - Inferring Genome Regulation Networks

Yeast cells → RNA measurements → Regulatory network

DNA → RNA → Protein

Contribution: formalise when two (causal) models at different levels of detail are consistent.

Transformations of SEMs

$\mathcal{M}_X$ implies the poset of distributions $\mathcal{P}_X := \{\mathcal{P}_X^{\omega(i)} : \omega(i) \in \mathcal{I}_X, \leq_X\}$

Suppose we are given $\mathcal{M}_X$ and a ’measuring device’ $\tau : \mathcal{X} \rightarrow \mathcal{Y}$

$X \sim P_X$ an r.v. in $\mathcal{X}$ $\Rightarrow \tau(X) \sim P_{\tau(X)}$ is an r.v. in $\mathcal{Y}$

$\tau : \mathcal{P}_X \rightarrow \mathcal{P}_{\tau(X)} = \{\mathcal{P}_{\tau(X)}^{\omega(i)} : \omega(i) \in \mathcal{I}_X, \leq_X\}$

Does there exist an $\mathcal{M}_Y$ such that $\mathcal{P}_X = \mathcal{P}_{\tau(X)}$?

Definition: Exact Transformations between SEMs

Let $\mathcal{M}_X$ and $\mathcal{M}_Y$ be SEMs and $\tau : \mathcal{X} \rightarrow \mathcal{Y}$ be a function. We say $\mathcal{M}_Y$ is an exact $\tau$-transformation of $\mathcal{M}_X$ if there exists a surjective order-preserving map $\omega : \mathcal{X} \rightarrow \mathcal{Y}$ such that

$$\mathcal{P}_{\tau(X)}^{\omega(i)} = \mathcal{P}_{\tau(X)}^{\omega(j)}$$

Examples of exact transformations

Stationary Behaviour of Dynamical Processes

Accepted to Uncertainty in Artificial Intelligence (UAI) 2017
https://arxiv.org/abs/1707.00819

Causal Consistency of Structural Equation Models

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An SEM $\mathcal{M}_X = (\mathcal{S}_X, \mathcal{Z}_X, \mathcal{P}_X)$ consists of

- $\mathcal{S}_X$ a set of structural equations $X_i = f_i(X_i, E_i)$
- $\mathcal{Z}_X$ is a distribution over the exogenous variables $E$.
- $\mathcal{Z}_X$ is a subset of all perfect interventions with partial ordering
- e.g. $\text{do}(X_2 = 0) \leq_X \text{do}(X_2 = 0, X_3 = 2) \leq_X \text{do}(X_2 = 0, X_3 = 0) \leq_X \text{do}(X_3 = 2)$

$X_1 = f_1(E_1)$
$X_2 = f_2(E_2)$
$X_3 = f_3(X_1, X_2, E_3)$
$X_4 = f_4(X_1, E_5)$
$E = P_E$