Payments networks and risk of firms

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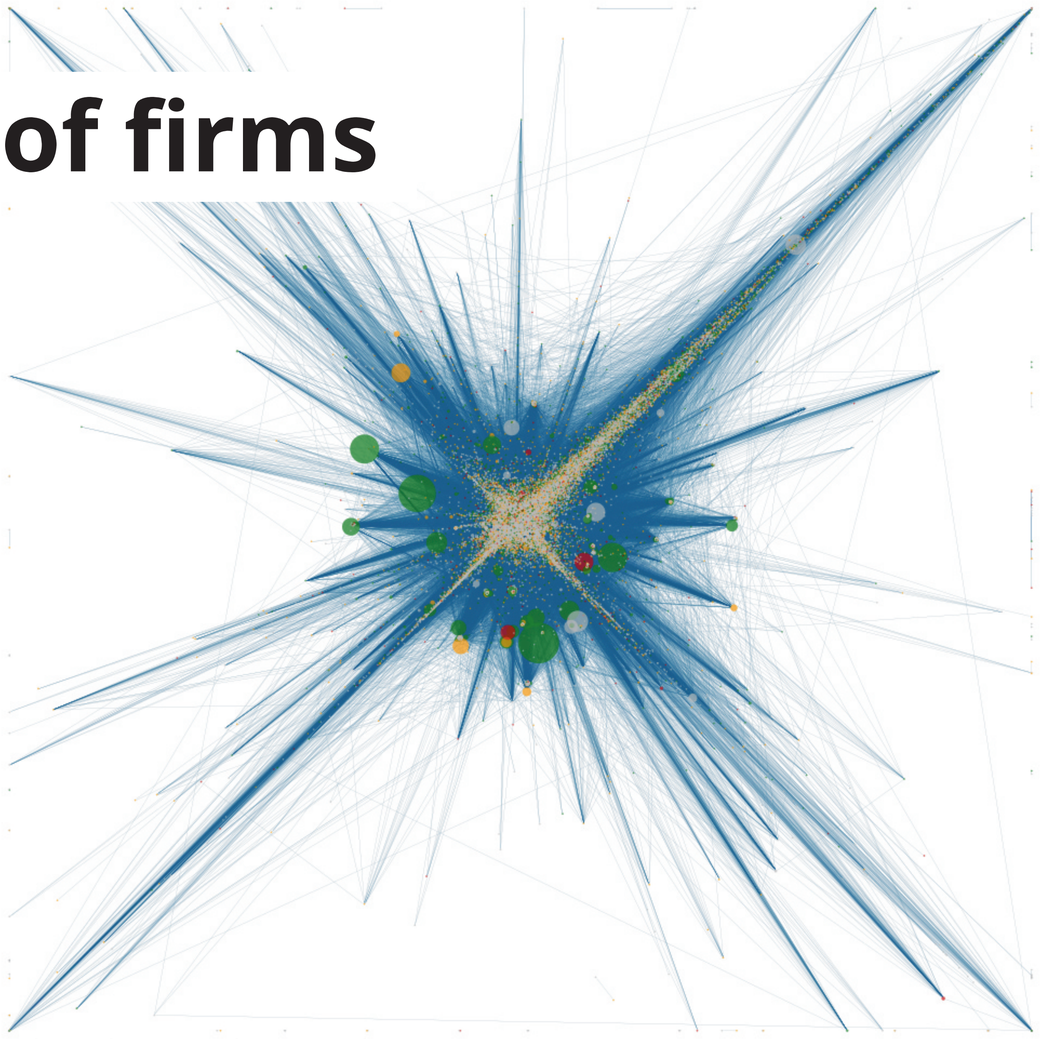


Representation of the payments network of firms. The size of the circle is proportional to the volume exchanged by the firm. The colour indicates the risk rating: green for low, yellow for medium, red for high risk firms

Motivation and contribution

Corporate firms **interact** with each other on a daily basis. The interactions can be of different kinds, including supplier-buyer relationships, business partnerships, financial contracts and ownership. At the same time, any firm has a **level of risk**, which may depend on many idiosyncratic factors as well as the industrial sector or its location. In the banking system, the riskiness of a client firm is quantified by an internal rating which maps to its probability of default. Even if the risk rating of a firm is not known to all the counterparts, it may affect the ability of the firm to engage in such interactions. For example, a poor rating may prevent the access to credit and as a result may cause a reduction or delay in payments toward suppliers.

These motivate the study of the system of firms from a **network perspective**. On one side, we would like to shift the attention from the single isolated firm to the system of interacting parts as a whole. On the other side, we want to understand whether and in which measure a firm's role in the network can be informative of its riskiness.

We study a large proprietary dataset of payments between Italian firms, which contains the payments collected between more than two million Italian firms together with information on risk rating for some of them. At first, we focus on standard network metrics, such as degree and strength distribution and components decomposition.

Then, the dependence between the network property of a firm and its risk is investigated considering the presence of correlation between node's local property and the risk; homophily of risk, i.e. the tendency to interact with firms with similar risk; and a localisation of a certain level of risk in network-induced communities of firms.

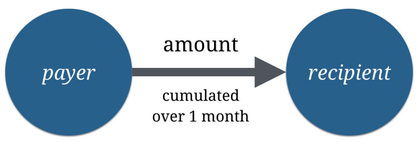
Networks of payments

The dataset - transactional data of the payment platform of a major Italian bank. Transactions are registered with daily granularity for the year 2014. Further information on the counterparts may be included.

Table: Dataset content

name	**entities**	**transactions**
entries	ID	ID source
	rating	ID recipient
	macro-sector	date (ymd)
	client status	type
		amount
		quantity
records	2 417 862	47 094 440

The networks - From transactions data, payment networks are constructed as follows: fixed a time window, each node represents a firm active in that period, if there is payment between two firms a link is put from the source to the recipient, with weight equal to the amount. If multiple transactions occur between the same (ordered) pair of nodes, the weight of the link is the sum of the amounts.



Each month the payments network of firms is very sparse but almost entirely made of a single weakly connected component (GC). Half of the firms appear in the network as payers only and they are mainly unclassified with respect to client status, so no much information is available on them. Of the remaining nodes, almost half constitutes the biggest strongly connected component (SCC), i.e. the denser core of the network where more than a half of transactions occur and above 60% of the volume circulates. Finally the network is small world, scale free, and slightly disassortative [Newman, 2002] both for degree and for strength.

Table: Average value over months for some basic metrics.

nodes	$1{\cdot}10^6$	power law (degree)	2.6
links	$3.2{\cdot}10^6$	power law (strength)	2.1
density	$3.01{\cdot}10^{-6}$	GC (% nodes)	98
assortativity	-0.03	SCC (%nodes)	20

Risk distribution

To measure the **dependence** between the network property of a node or a group of nodes and the risk of the firm represented by the node(s), we proceed zooming out from single nodes to subsets. At first we consider a firm's local property (the number of connections) and we check if it correlates with the risk. Then we consider pair of linked firms and measure the homophily in risk, i.e. whether firms with similar risk profile tend to do business together and thus to be linked. Finally, we divide firms into subsets induced by the network structure and we check whether the inferred subsets are informative with respect to the riskiness of the composing firms.

Single nodes - Small degree nodes are more likely medium risk firms, whereas large degree nodes are more likely low risk firms. The high risk firms are more evenly spread across degrees, even if a larger fraction is observed for low degree nodes.

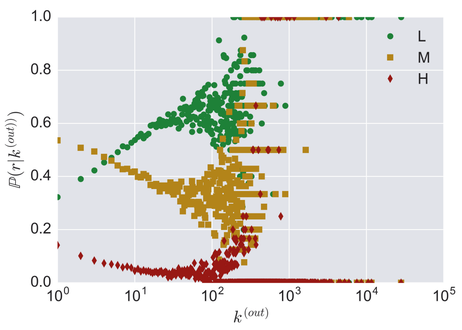


Figure: Probability of rating of a firm conditional to its out-degree

Pairs of nodes - A weighed version of assortativity is employed and the results indicates a **positive assortativity of risk** for both the entire graph, and the subgraph of clients, i.e firms tend to interact more with other firms with similar rating.

Communities - Two types of organisation of nodes into groups are considered: **modular** [Blondel et al., 2008] and **hierarchical** [Gupte et al., 2011].

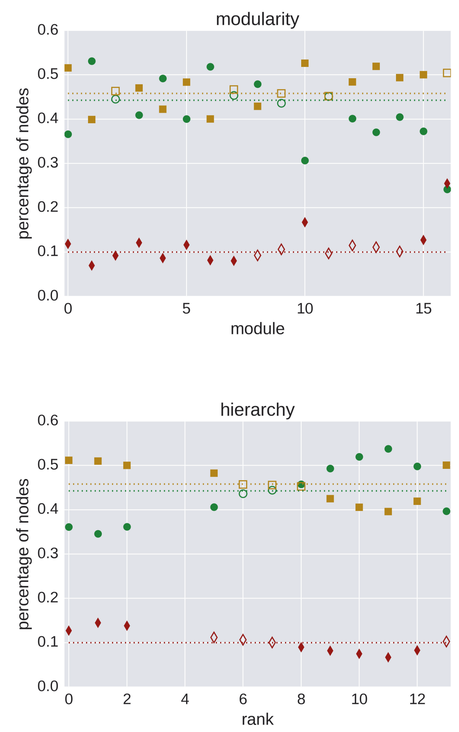


Figure: Distribution of ratings in the two partitions, modularity (top), hierarchy (bottom). The dashed lines are the unconditional (null) distribution of ratings among nodes in the entire sample. A full marker indicates that the over or under representation with respect to the null distribution is statistically significant in the hyper-geometric test at 1% significance level with Bonferroni correction. Over- (under-) expression are symbols above (below) the dashed line.

In the first case, each module is composed by nodes, which are much more connected among themselves than with the rest of the network. In the second case, we look for an ordered partition such that most links is from nodes of low rank classes to nodes in high rank classes.

To get more reliable results, we filter the graph, keeping only the clients.

For each metric, we proceed in the following way:

i. find the optimal partition
ii. compute the distribution of ratings within each subset of the partition
iii. test if the local distribution is statistically different from the overall distribution of ratings

Conclusion

The study of the structure of the network highlight a **complex interdependence** between firms; particularly interesting is the presence of a relatively small core of firms, which are involved in most transactions. This feature paired with the power-law tail distribution of the number of connection and the total volume exchanged by firms, can be symptom of an architecture which favours the spread of distress, or positive feedbacks. Also relevant, is the observed tendency of big, well-connected firms to be connected to small (in terms of exchanged volume), poorly connected firms. This can be the result of almost exclusive relationship between a big producer and its subsidiaries.

From our analysis of the correlation between the network structure and the distribution of risk, we can conclude that **the risk level of a firm can affect its features and role in the network at different levels**. For single firms, we observed that low risk firms are more likely to have a high number of connections, and some of them acts as hubs for the entire network, being connected to thousands of other firms. This feature is reassuring from a risk management perspective, as the most connected are also the less risky. When pair of linked firms are considered, we observed the tendency to favour connections towards same risk level firms. This tendency can be observed also on a more aggregate level. Indeed, we found that also groups of firms which are more connected among them that the rest of the network, have a local distribution of risk which is statistically different from the global one, meaning that some risk classes are over- or under- represented. Finally, we divided firms into a hierarchical organisation, in such a way to highlight the main direction along which money circulates. This simplified structure showed once more that many levels of the hierarchy have a local distribution of risk statistically different from the global one. As high risk firms are over-represented at the beginning of the flow of money, this can be a source of distress for the entire system.

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